ANGLE CALCULATIONS

A. Introduction

This document has been prepared to explain how to calculate the optimal angles to tilt the solar array and park the car at to maximize energy output over a given time period. The time period in question may be the designated static charging periods at the Formula Sun Grand Prix competition, or any time of day the vehicle is taken off the track to recharge.

B. Assumptions

One assumption made to simplify these calculations is that the solar array on the car is a flat panel. Effects of having the cells at different angles on the top body are not accounted for. The second major assumption made is that there is no shading from the driver’s canopy over the solar cells.

C. Variable Definitions*

- θ_z Zenith angle: the angle between the incident beam radiation and the normal to a horizontal surface (Fig. 1).
- γ Surface azimuth angle: the angle between the local meridian and the projection of the normal to the solar cells on a horizontal surface (Fig. 1). West is positive, East is negative, and angle measurements are taken with respect to South. $0^\circ \leq \gamma \leq 180^\circ$.
- γ_s Solar azimuth angle: the angle between South and the projection of the incident beam radiation on a horizontal surface (Fig. 1). Here, West is positive, East is negative, and angle measurements are taken with respect to South. $0^\circ \leq \gamma_s \leq 180^\circ$.

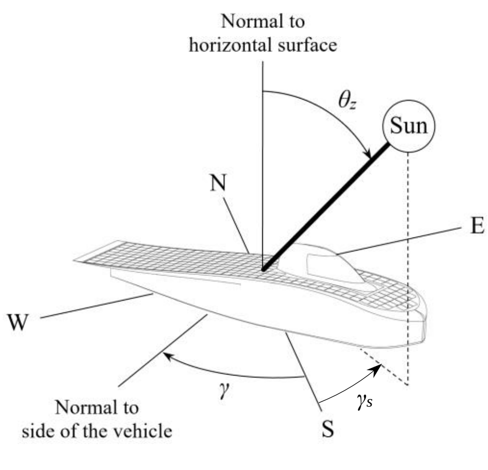

Fig. 1. Diagram depicting the zenith angle, surface azimuth angle, and solar azimuth angle.

- β Slope: the angle of inclination of the solar cells with respect to the plane of a horizontal surface (Fig. 2).
- θ Angle of incidence: the angle between the incident beam radiation and the normal to the solar cells (Fig. 2).

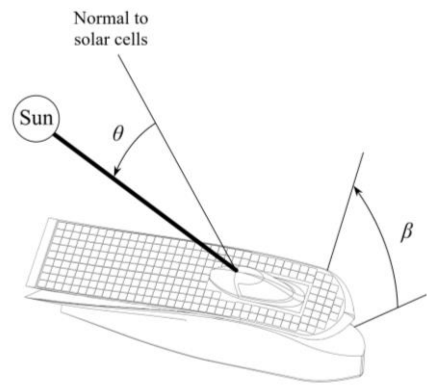

Fig. 2. Diagram depicting the slope and angle of incidence.

- δ Declination: the angle between the incident beam radiation and the plane of the equator on a given day of the year (Fig. 3). $-23.45^\circ \leq \delta \leq 23.45^\circ$.
- ω Hour angle: the angular displacement of the sun East or West of the local meridian due to the Earth’s rotation on its axis (Fig. 3). The hour angle is negative before solar noon and positive after solar noon. $-180^\circ \leq \omega \leq 180^\circ$.

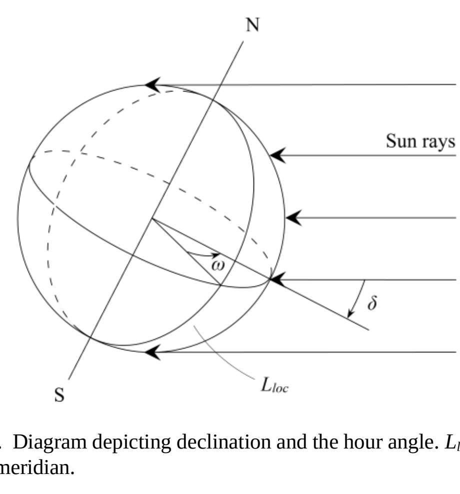

Fig. 3. Diagram depicting declination and the hour angle. L_{loc} is the local meridian.

D. Objective

The two angles we are seeking to optimize are γ and β. If turns out that for a given time period, the

*All trigonometric angle calculations in this document are performed in degrees.

optimal value of γ is the average value of γ_s and the optimal value of β is the average value of θ_z over that time period. It is likely that the car will only be parked once per charging period, and the following math will therefore yield just one optimal value of γ. If it is determined that the solar array will only be angled once per charging period, only one optimal value of β will be calculated. However, if it is desirable to change the angle of the solar array multiple times per charging period, say every 30 minutes, then the optimal values of β simply become the average value of θ_z over each 30 minute interval. As an extension of this concept, a single axis tracking system, the best case scenario, would have $\beta = \theta_z$ at all times.

Note that because of the assumptions made, it may be necessary in practice to make adjustments if shadows are observed, or monitor current with a multimeter while the angle is being changed to find where it is maximum.

E. Calculating β from θ_z

Midpoint Time

To calculate the optimal slope β of the solar array for a period between any start time t_1 and end time t_2, begin by finding the midpoint time t with the equation

$$t = \frac{t_1 + t_2}{2} \tag{1}$$

Here, t is in local standard time – the time that appears on a clock. If the calculations are performed with a computer code, it may be necessary to convert the times to numerical values representing the clock times to perform mathematical operations on them. If t_2 is not known, an estimate will be required.

Conversion from Standard Time to Solar Time

The midpoint time of each charging period must be converted from local standard time to solar time. Solar time is in reference to solar noon – the time at which the sun reaches the greatest altitude of the day. First, the equation of time E is evaluated with the equation

$$\begin{aligned} E = 229.2(&0.000075 + 0.001868\cos(B) \\ &- 0.032077\sin(B) - 0.014615\cos(2B) \\ &- 0.04089 sin(2B)) \end{aligned} \tag{2}$$

where the constant B is given by

$$B = (n-1)\frac{360}{365} \tag{3}$$

and n is the day of the year. The conversion to solar time t_{sol} is performed with the equation

$$t_{sol} = t + 4(L_{st} - L_{loc}) + E \tag{4}$$

where L_{st} is the standard meridian for the local time zone, and L_{loc} is the longitude of the location at which the performance of the solar cells was evaluated. Note that the units of E and the second term in this equation are minutes. Finally, to account for the one hour time difference in locations that observe daylight saving time (DST), a corrected solar time t_{sol}’ is calculated:

$$t_{sol}' = \begin{cases} t_{sol} & 1 \le n < 68 \\ t_{sol} - 60, & 68 \le n \le 306 \\ t_{sol} & 306 < n < 366 \end{cases} \tag{5}$$

The “60” represents 60 minutes. In the year 2020, a leap year, DST is observed from the 68th day of the year until the 306th day of the year.

Zenith Angle

To estimate the beam and diffuse components of incident radiation, the declination angle δ was first calculated with the equation

$$\delta = 23.45 sin\left(\frac{360(284+n)}{365}\right). \tag{6}$$

Next, the hour angles at the midpoint time t was calculated. The hour angle is the angular displacement of the sun east or west of the local meridian, due to the rotation of the earth. Since the earth rotates about its axis at a rate of 15° per hour, the hour angle ω is calculated with the equation

$$\omega = 15 \cdot t_h \tag{7}$$

where the variable t_h represents the time difference between the current solar time t_{sol}’ and solar noon in hours. The variable t_h is negative if the current solar time is before solar noon and positive if the current solar time is after solar noon. Since t_h has been calculated from the midpoint time t, then ω is the midpoint, or average, hour angle.

Then, the average zenith angle θ_z for the time period is calculated with the equation

$$cos\theta_z = cos\phi cos\delta cos\omega + sin\phi sin\delta \tag{8}$$

Finally, the optimal value of β is equal to θ_z, or

$$\beta = \theta_z \tag{9}$$

Calculating γ from γ_s

Pseudo Solar Azimuth Angle

To calculate the optimal surface azimuth angle γ, it is necessary to know which quadrant of the cardinal directions plane the sun is in. Therefore, first calculate a pseudo solar azimuth angle γ_s' in the first or fourth quadrant, then use multiplicative factors to determine the solar azimuth angle γ_s at the midpoint of the charging time. First, the pseudo solar azimuth angle γ_s' is found from the equation

$$sin\,\gamma_s' = \frac{sin\omega cos\delta}{sin\theta_z} \tag{10}$$

Solar Azimuth Angle

Next, calculate the hour angle at which the sun is due west (or east), denoted as ω_{ew}. This hour angle can be determined from the equation

$$cos\,\omega_{ew} = \frac{tan\delta}{tan\phi} \tag{11}$$

Now, the solar azimuth angle γ_s can be calculated with the equation

$$\gamma_s = C_1 C_2\,\gamma_s' + C_3\left(\frac{1 - C_1 C_2}{2}\right)\cdot 180 \tag{12}$$

where

$$C_1 = \begin{cases} 1\ if\ |\omega| < \omega_{ew} \\ -1\ otherwise \end{cases} \tag{13}$$

$$C_2 = \begin{cases} 1\ if\ \phi(\phi - \delta) \geq 0 \\ -1\ otherwise \end{cases} \tag{14}$$

$$C_3 = \begin{cases} 1\ if\ \omega \geq 0 \\ -1\ otherwise \end{cases} \tag{15}$$

and ω is still the average hour angle. The optimal surface azimuth angle γ is then equal to the average solar azimuth angle γ_s

$$\gamma = \gamma_s \tag{16}$$

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